Name: Feedback

SA402 · Dynamic and Stochastic Models Fall 2023 · Uhan

Quiz 2 – 8/30/2023

Instructions. You have 15 minutes to complete this quiz. You may use your plebe-issue TI-36X Pro calculator. You may not use any other materials.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	0.5	
1b	0.5	
2	1	
Total		/ 20

Problem 1. Consider a Markov chain with state space $\mathcal{M} = \{1, 2\}$, and transition probability matrix $\mathbf{P}$ and initial state vector $\mathbf{q}$ as follows:

$$\mathbf{P} = \begin{bmatrix} 0.6 & 0.4 \\ 0.8 & 0.2 \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix}$$

a. What is $\Pr\{S_1 = 2 \,|\, S_0 = 1\}$?

See Problem 1a in the Lesson 3 Exercises for a similar (actually, the same) example.

Some of you overthought this problem. You should be able to answer this problem by simply looking at $\mathbf{P}$.

b. What is $\Pr\{S_2 = 1 \text{ and } S_1 = 1 \text{ and } S_0 = 2\}$?

See Example 1 in Lesson 3 for a similar example.

Note that this problem is not exactly the same as Problem 1b in the Lesson 3 Exercises. In this problem, the probability you are being asked to compute is not conditional on $S_0 = 2$.

Problem 2. An automated guided vehicle (AGV) transports parts between three locations: a release station A, a machining station B, and an output buffer C. The movement of the AGV can be described as making trips from location to location based on requests to move parts. More specifically:

- If the AGV is at the release station A, it moves next to the machining station B.
- If the AGV is at the machining station B, it is equally likely to move next to either of the other two locations.
- If the AGV is at the output buffer C, it is equally likely to move next to either of the other two locations.

Suppose at the beginning of the day, the AGV is equally likely to be at any of the three stations.

Model this setting as a Markov chain. Specify the transition probabilities as a matrix.

See Problem 1 from the Lesson 4 Exercises for a similar example.

Don't forget to specify all the parts of a Markov chain model! Recall from page 1 of Lesson 4: to specify a Markov chain model, we need to describe

(1) the state space and the meaning of each state in the setting's context
(2) the meaning of one time step in the setting's context
(3) the transition probabilities and initial state probabilities

Some of you forgot the highlighted parts above.